3rd Grade Mathematics Lesson Plan

"Multiplication Algorithm (1)"

December 6, 2006 (Wednesday)
1:35 p.m. ~ 2:20 p.m.

Sapporo City Maruyama Elementary School
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○ The goal of the Mathematics Group at Maruyama is to develop students' ability to use what they learned before to solve problems in the new learning situations by making connections

○ In addition, we want to provide 3rd grade students with experiences in mathematics that enable them to use what they learned before to solve problems in new learning situations by making connections.

○ This lesson, "The Multiplication Algorithm (1)," is designed to utilize prior learning to make connections and solve problems in new learning situations.

- Goals of the Unit:
- Lessons that enable students to consciously think about the connection between what they learned before and what they are learning now
- Lessons in which students learn from each other and that help them consciously think about their own solution processes
- An evaluation method that helps foster students' logical thinking abilities.
- Unit plan
- This lesson (goals, process of lesson

Mathematics Group at Maruyama

Developing students who can use what they learned before to solve problems in new learning situations by making connections

1. Goals of mathematics group

In the course of study, the objectives of arithmetic (elementary mathematics grades 1 to 6) are indicated below:

"To help children acquire basic knowledge and technical skills with respect to numbers, quantities, and geometric figures <u>through mathematical activities</u>; to foster <u>the ability to think insightfully and logically</u>; and to foster the attitude to enjoy the activity and appreciate the merit of mathematical manipulation, and to willingly make use of it in day life."

2. The actual state of students at Maruyama

When we analyzed the achievement test scores of the students at our school, we found that our students are way above the scores of the national average in every domain of arithmetic, although the drop in student achievement is becoming a topic of discussion in Japan.

3. The key issue at Maruyama

Although the students at Maruyama seem to be doing well, as reported above, we feel that there are many students who are just waiting to receive instructions on how to solve problems instead of attacking new and challenging problems. We think that there are not many students who show a strong desire to tackle very challenging problems by saying, "I want to solve this problem on my own, even if it takes a long time." Also, there are not many students who enjoy solving problems by trial and error.

We believe that this is a result of lessons that have not been providing enjoyable experiences where students reach solutions on their own, see interesting regularities or patterns in the investigations, and think about and share the questions that come up from the learning with their classmates.

In order to develop students who can feel the enjoyment of mathematical learning and acquire the logical thinking skills that are stated in the course of study, we decided to develop lessons with the following point of view.

4. What kinds of lessons develop students who can use what they learned before to solve problems in new learning situations by making connections?

(1) Contents of the subject

We believe that promoting problem solving through mathematical activities will help us to achieve this objective.

Developing learning with clear systematic content connections

We believe that teachers need to be more clear about how the topics that students study are connected to each other. We need to think about "how students can use previously learned content to solve problems in new situations" or "how different problem solving situations require different kinds of prior learning," and incorporate these ideas into the development of units and lessons.

Helping students to become more conscious about the process of their own solution

We believe that students should be more aware of their own solution processes, be able to articulate how they made a connection between prior learning, and how they used the idea to solve problems in new situations.

Students acquire the ability to think on their own by drawing diagrams and number lines, reflecting on their own solution processes, determining what they do understand and what they don't understand, and comparing their own solution with those of their classmates.

We believe that students should not only focus on the rightness or wrongness of their answers, but should also reflect on their own solution processes and understand that it is important to feel the real enjoyment of mathematical learning in addition to getting right or wrong answers.

Thinking about evaluations that help students to become particular about the solution process

We need to think about what the points we need to pay attention to are to evaluate student learning in order to help students to develop the "**mathematical thinking**" necessary to conduct meaningful and effective problem solving activities.

(2) Important points of view of mathematics group

Considering the actual state of learning of the students at Maruyama and the content of the above subject, we think that it is important to develop units and lessons with the following three points of view in order to achieve the objective of "developing students who can use what they learned before to solve problems in new learning situations by making connections."

[Point of view #1]
"Develop instructional materials that pay attention to the connections between previously learned content and new content"

> "It is important to clarify the mathematical thinking that students learn in 6 elementary school years by investigating the instructional materials and developmental processes of students. To do so, we need to understand how previously learned content is necessary to learn new content, and how useful it is."

> (1) Connections between grade levels
> (2) Connections within the domain
> (3) Connection between self and problems

What students learned about multiplication in 2nd grade is not just useful for calculation. The idea they learned about "how many times a quantity is as a unit quantity", rather, is a fundamental idea in mathematics.

Furthermore, in 2nd grade students learn "length" by direct comparison, indirect comparison, and measurement with arbitrary units. Afterwards, the students who recognize the necessity of measuring with a universal unit can also learn "weight" in 3rd grade by using similar thinking.

Students who think about the "why" in the process of problem solving can begin making connections between the problem and what they need to think about, as well as how they need to think.

[Point of view 2]
"Students can learn from each other and this will help them to consciously think about their own solution processes."

There are many of new things that students can learn from each other by learning during lessons in classroom when they actually feel the merit of the mathematics and beauty and value of it.

> ○ Students who can clarify their own solution processes and participate in the discussion to learn from each other
> ○ Students who can learn through discovery by comparing their own thinking and that of others.
> ○ Students who can reflect and evaluate what they do and don't understand
> ○ Students who are particular about how they solve problems

Classroom learning experiences that foster learning from each other as explained above not only enhance student learning but also develop a strong tie among the students.

[Point of view 3]
"Evaluation that foster students' logical thinking ability"

In order for the students to be able to "think logically," we believe that they need to be particular about their own solution processes when engaged in problem solving activities.

> Teacher's support for enhancing students' learning from each other

First of all, in order for students to enhance their learning by learning from each other, we thought that it very important to provide teacher support by organizing the blackboard and highlighting important points of the lesson to enhance student thinking.

Secondly, we wanted to plan appropriate supports so that students need feel like they need to think about what prior knowledge they need to recall and can make connections to the new problem situation. In addition, we wanted to include supporting questions to encourage students to think about their solution processes deeply, understand each other' ideas, including the similarity and differences in these ideas, to deepen the understanding they can gain from one another. Lastly, we prepared a second problem that helps us to understand students' learning during the lesson to support the understanding of the effect of students learning from each other in the lesson.

Teacher's support for enhancing students' learning from each other

We will administer tests in order to understand students' actual state of learning. Testing is not the only way to understand students' actual state of learning but it can be useful if teachers can use it to reflect and improve their own teaching and classroom lessons conducted.

Fostering students' expressive mathematical ability

It is important to foster students' expressive mathematical abilities by learning from each other in the classroom. Some of the examples of the abilities that we want to foster are:

- ○ Being able to describe ideas using number lines and diagrams
- ○ Being able to manipulate concrete materials and explain ideas to others
- ○ Being able to think about and understand the meaning of numbers and operations as expressed in math sentences
- ○ Being able to take notes that reflect the students' thinking and points of view

Students who acquire this kind of ability can participate in classroom learning in following ways:

At the end of 2nd grade, students begin using expressions such as "because…" to describe their reasons and support their ideas.

In 3rd grade, they begin comparing their own ideas with others and use expressions such as "my idea is similar to so-and-so's idea…"

In 4th grade, students use expression such as "for example…" and "because…," more frequently Moreover, they begin to use hypothetical statements such as "if it is… then…"

In 5th grade, they can become more sophisticated and make statements such as, "If it is … then it will be □, but if it is ◆ then I think we can say ✶" by looking at different conditions.

Finally, in 6th grade, students can start describing in ways such as, "It can be said when it is … but in this situation □ is much better," and starting to make decisions about how to choose a better idea.

We are hoping to see this expressive mathematical ability more often in the classroom, therefore, we would like to examine students' actual state of learning more carefully.

(3) What is important to the Mathematics group?

Making a better communication with other subject area groups

It is important to for the Mathematics group to establish good communication with other subject groups at the school.

Our value and view of classroom teaching and learning described before should be understood by all staff members so we can provide a consistent and systematic approach to educating our students in the school as a whole.

In order to foster "assured abilities" and "enriched hearts and minds"

We believe that "feelings and emotions" need to be involved in student learning. The "feelings and emotions" that we are talking about here are the "hopes," "desires" and "particularity" that are necessary for students to autonomously and actively be involved in their own learning. This includes the feelings and emotions expressed through words such as "I wonder why?" "If it is … then …" "I wonder if it is always true…," and, "I found this discovery…"

These are some of the things we hope to and are striving to achieve. We believe that "**the knowledge gained though feelings and emotions**" will truly help the students to acquire "assured abilities" and "enriched hearts and minds."

3rd Grade Mathematics Lesson Plan "The Multiplication Algorithm (1)"

Date: December 6, 2006
Instructor: Hideyuki Muramoto
Classroom: 3rd grade, class B (39 students)

1. Theme: Third grade mathematics lessons that foster students' ability to use what they learned before to solve problems and make connections in order to solve problems in new learning situations

We conducted a survey about mathematics learning with 3rd grade students at Maruyama elementary school.

Do you like mathematics?

Yes ------------------------ 50%
Somewhat yes ---------- 44 %
Somewhat no ------------ 5%
No ------------------------- 1%

Students that said "yes"

- I like calculations and enjoy them.
- If I understand, it is fun.
- Because answers are clear.
- Because I can listen to everybody's different ideas.

Students that said "somewhat yes"

- I like calculations, but I don't like story problems.
- It is very difficult to construct math sentences for story problems.

Students that said "somewhat no" and "no"

- I don't like story problems.
- Exams are difficult.
- It is very difficult to construct math sentences.

The 3rd grade students at Maruyama like calculations but many of them feel that they are not good at being able to construct math sentences for story problems. Therefore, the following ideas were used to develop units and lessons.

We want to increase the number of students who like thinking logically and equip them with the skills they need to understand the structures of story problems by using diagrams and number lines.

We want to increase the number of students who are interested in listening to other students' solution processes, think about "whether the solution process is similar or different", and be able to communicate.

The topics students learn in the 3rd grade are as follows:

(1) Addition & subtraction (3-digit numbers)
(2) Multiplication (2-digit & 3 digit by 1-digit by using the algorithm)
(3) Division (meaning, remainder)
(4) Large numbers (up to 10 million)
(5) Time and elapsed time (meaning)
(6) Capacity, length, weight
(7) Characteristics of rectangles and squares
(8) Shapes of boxes (characteristics & nets)
(9) Tables and bar graphs (categorizing data and constructing tables and bar graphs)

One of the most important mathematical ideas that students learn in almost all of the domains in 3rd grade mathematics is thinking about quantities in terms of "how many times as much it is as a unit quantity." For example,

○ In addition, subtraction, and large numbers, we look at 1, 10, 100, etc, as unit

○ In multiplication and division, we look at "how many times as much as a unit" and "dividing something into a number of units"

○ In time and elapsed time, capacity, length, and weight, we look at "how many times as much as a unit"

By using the big mathematical idea of "how many times as much one quantity is as a unit quantity" as an umbrella, we can develop lessons that help emphasize this idea as well as think about everyday lessons that will help nurture this idea by, for example:

- Developing lessons that help students to become aware of the connection between what they learned before and what they are learning now and use previously learned knowledge to overcome obstacles in a new situation. (Connections between previously learned knowledge and new learning)
- Representing a problem situation with diagrams based on the idea of "how many times as much as a unit quantity" consistently and helping students to understand the situation and solution of the problem more clearly; and developing lessons that incorporate this idea and help students to use the diagram to think

about the solution of a problem logically. (Logical thinking)

- Developing lessons that help students to understand what they need to compare with. This understanding will be enhanced with lessons that pay attention to the process of solution methods that utilize previously learned knowledge. How to express the process as part of the solution and student presentation are also important. (Solution process)

We would like to help the students to be aware of the importance of the idea of "how many times as much as a unit quantity"

2. Teaching "the Multiplication algorithm (1)" in a way that develops students who can use what they learned before to solve problems in new learning situations by making connections

Goals of the Unit

- To try to think about how to calculate 2- or 3-digit x 1digit numbers by using the ideas about multiplication that they learned previously (2- or 3-digit x 1-digit number calculation by utilizing the idea of decomposing numbers or the base 10 decimal system)
- Be able to do 2- or 3-digit x 1digit number calculation using algorithm

About the Instructional Material:

The list below shows the content that students learned prior to this unit:

- 1-digit x 1-digit multiplication (grade 2)
- Multiplication involving 0, multiplication of 10's (grade 3)
- Using the idea of the distributive property of multiplication to create the multiplication table (e.g., the multiplication table of 7 can be developed by using the tables of 5 and 2.)

The purpose of this lesson is to help students think about how to multiply 2-digit numbers by 1-digit numbers. By just looking at the math sentence (e.g., 23 x 3), many students feel that the problem cannot be solved by using the multiplication tables. If students can see the structure of the problem with an array diagram, however, they can notice that they can calculate this problem by using the multiplication facts that they already learned. I want to make sure that students can see that they can utilize the idea of "how many times as much as a unit quantity" in this case also.

In this lesson, students will decompose the 2-digit number into numbers that are easy to use the multiplication tables with. Through this investigation, students will realize the merit of decomposing the 2-digit number into tens and ones (e.g., 23 → 20 and 3) to do the calculation. In addition, they will learn that this idea is the basis for the process of the multiplication algorithm (pencil and paper calculation method).

A main point of this lesson is for students to investigate how to decompose the number 23 so they can use their previously learned knowledge. It is important that students understand the merit of decomposing 23 into 20 and 3 to in order to understand the mechanism of the algorithm. In this lesson, however, I would like students to look at an array diagram to recognize the merit of decomposing the 2-digit number so they can use the multiplication table to calculate.

Learning from each other by paying attention to the solution process

Since the beginning of the school year (April 2006), I have been teaching students to draw a diagram of the problem situation in order to think about the solution. In addition, I have been emphasizing the importance of mathematical learning in class so students are able to use the diagram to explain their thinking processes logically.

There are some students in the class who already know how to do multiplication using the algorithm. Although, many of them know the calculation procedure, it is not clear if they really understand the mechanism of the calculation. The students can understand this, however, by looking at an array diagram, recognizing the meaning and merit of decomposing the 2-digit number to calculate, and generalizing the idea of "how many times as much as a unit quantity."

The solution to the problem 23 x 3 is always 69 regardless how you decompose the number 23 to do the calculation. Students will realize how different ideas for the calculation can be used by learning from each other in the classroom.

Students that are hesitant or experience difficulty with 2-digit multiplication may not be able to come up with the idea of decomposing the 2-digit number and instead may think about using addition (23 + 23 + 23 = 69). By learning from each other in the classroom and being exposed to different ideas, however, they may start thinking, "If I decompose 23, I wonder if the calculation will become easier…"

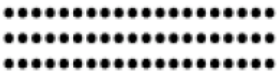
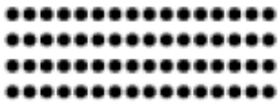
An array diagram that shows how the number 23 was decomposed differently and the math sentences that go with each different method will help these students to compare ideas and think about a better method.

Evaluation points for fostering students' logical thinking

In order to foster students' logical thinking ability, I would like to pay attention to the following points and help students to recognize them individually and as whole class.

- ○ Are the students using diagrams to understand the problem situation?
- ○ Can the students show their own thinking using the diagrams?
- ○ Can they reflect, justify, or analyze their thinking by using the diagrams?
- ○ Can they express their thinking or thinking process by using words like "because…"therefore…" "for example…" "if it is… then…" and "since… then…"
- ○ What point of view do the students use to compare the different ideas?
  - o How the answers are different
  - o How the expressions are different
  - o The reasons behind their thinking
  - o How previously learned knowledge is used
- ○ Can they recognize the point and merit of comparing different thinking and the new questions that result from the comparison?
- ○ Can the questions that evolve in the classroom students to their own ideas?

Unit Plan
(Total 13 lessons, this lesson is the 4th lesson)

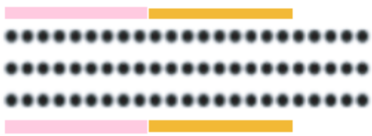
	Learning Activities
1	 How many ● are there? Let's find out by calculation! o Because we have 3 groups of 20 dots… o I wonder if we can use the multiplication facts to do the calculation… o 20 x 3 20 + 20 + 20 o 20 is two 10s. o We can find out how many 10's are there by using 2 x 3.
2	Let's think about story problems that show the math sentence 20 x 3 o "Each chocolate costs 20 yen. We bought 3 of them. What is the total price?"
3	If the price of one item is 300 yen, then what is the math sentence? o 300 x 3 o This time we can think about how many groups of 100 there are. o We can find out how many 100's are there by using 3 x 5.
4	 How many ● are there? Let's find out by calculation! o This time one group is 23 dots. o It is about 60 dots. o The math sentence would be 23 x 3. o We can't easily calculate this using multiplication facts. o If we divide 23 into smaller parts then it looks like we can use multiplication facts. o We can use an algorithm (paper and pencil calculation menthod) to calculate. o 9 x 3, 9 x 3, 5 x 3 Altogether, it becomes 69. o 10 x 3, 10 x 3, 5 x 3 Altogether, it becomes 69. o 20 x 3, 3 x 3 Altogether, it becomes 69. o Which one of these ideas is easier to calculate? o All of the ideas use the idea of dividing 23 into smaller parts.
5	Let's think about how to calculate using the algorithm (paper and pencil calculation method)! $\begin{array}{r} 23 \\ \times\ 3 \\ \hline 69 \end{array}$ o Think about 23 as 20 and 3. o Putting together 3 x 3 and 20 x 3 o We can do the calculation using multiplication facts
6	 How many ● are there? Let's find out by calculation! o The math sentence is 16 x 4. o It should be more than 40. It looks like it's a lot more than 40. o We can do this calculation by dividing 16 into 10 and 6 just like we did before. Let's do this calculation by using the algorithm. o 6 x 4 = 24 We can't write 24 in the ones place. I wonder how I should write the number... o We can write the 2 of the 24 in the tens place.
7	Let's make many problems of □□ X □! Let's think about everybody's problems using the algorithm. o Some of the answers become 3-digit numbers. o There are answers where the tens place becomes 0. o There are problems that involve regrouping twice.
8	The price of 1 *m* ribbon is 312 yen. We bought 3 *m* of ribbon. How much did the ribbon cost? o What would the estimated answer be? o It should be more than 900 yen. o The math sentence is 312 x 3. I wonder if we can use the algorithm again for this one... o If we divide 312 into smaller parts, we can calculate this. o 300 x 3 10 x 3 2 x 3 Altogether, it becomes 936.
9	Let's make many problems of □□□ X □! o I made a problem where the answer becomes a 4-digit numer. o I made a problem that involves regrouping.
10	Let's practice calculating with the algorithm!
11	The price of a piece of cake is 60 yen. There are 4 pieces of cake in each box. If we buy 2 boxes, how much will the total price be? o I think we need two math sentences to solve this problem. o First, find out the price for 1 box. 60 x 4 = 240 We have 2 boxes of 240 yen. 240 x 2 = 480 o First, find out total number of cakes. 4 x 2 = 8 One piece of cake is 60 yen, so 60 x 8 = 480 o We can begin calculating either way.
12	Let's pracitice!
13	Let's review what we learned in this unit!

3. The learning of this lesson

Goals:

- To be able to think about how to carry out the calculation of a 2-digit number x a 1-digit number by using what was previously learned about multiplication (mathematical thinking)

The process of the lesson:

Learning activities and students' anticipated reactions and thinking	Points to remember
 How many ● are there? Let's find out by calculation! • There are 23 dots in a row. • There are 3 groups of 23 dots. • There are more than 60 dots. • We can find out the number of dots by counting or by addition. • I wonder if we can use what we learned before about multiplication. • The math sentence would be 23 x 3. We can't simply use the multiplication table to do the calculation. What should do we do?  Divide 23 into 9, 9, and 5 9 x 3 = 27 9 x 3 = 27 5 x 3 = 15 27 + 27 + 15 = 69 69 dots  Divide 23 into 10, 10, and 3 10 x 3 = 30 10 x 3 = 30 3 x 3 = 9 30 + 30 + 9 = 69 69 dots  Divide 23 into 20 and 3 20 x 3 = 60 3 x 3 = 9 60 + 9 = 69 69 dots If we divide 23 into smaller parts, we can use many different multiplication facts from the multiplication table to do the calculation. Which one of them do you think is a good idea? What are the similarities or differences among the different solutions? • All of the methods decided to divide 23 into smaller parts. • There are methods that involve dividing 23 into 3 parts and 2 parts. • The numbers used in the math sentences are different. • If we use the multiplication 20 x 3 that we learned previously, we have two math sentences.	In order to understand the problem task, help students to see the dots as "how many in a group" and "how many groups." Before doing the actual calculation, encourage students to estimate the answer. Praise the idea of recalling what the students learned before. Try to understand students' different ideas by walking around the classroom. When you find students that are solving the problem with addition, ask them, "Can you use multiplication for this calculation?" Make sure to use the diagrams to represent how the calculations were done. When students are learning each other's calculation methods through the students' presentations, make sure that they consider how the methods are related. Make sure to encourage the students to compare the different ideas and help them to make a conscious effort to make their own value judgments about the different ideas. If I find a child is using an algorithm to do the calculation, I would like to ask the child to think about how the calculation

• I used an algorithm (paper and pencil calculation) to do the calculation $\begin{array}{r} 23 \\ \times\ 3 \\ \hline 69 \end{array}$ If we compare this method and the array diagram… • This method also divides 23 into 20 and 3. If we divide a number into some small parts so we can use the multiplication facts, we can do the calculation in today's problem. The idea we used in the algorithm (paper and pencil calculation) is similar to the idea of dividing 23 into 20 (2 in the tens place) and 3 (3 in the ones place).	method is related to the array diagram. Make sure to highlight the idea of "make the calculation easier by using the multiplication table and the other kinds of multiplication the students learned before." Also, if a child shows the algorithm calculation method, then help him/her to consciously connect the idea with this idea.